Analysis of sandwich beams using mixed finite element

S. BOUZIANE[a], H. BOUZERD[a] and M. GUENFOUD[b]

a. *Civil Engineering Department, University of Skikda, Skikda, Algeria*
b. *Laboratory of Civil Engineering and Hydraulic, University of Guelma, Guelma, Algeria*

Résumé:
Un élément fini mixte d'interface basé sur le principe variationnel mixte de Reissner est présenté pour analyser les poutres sandwich. C'est un élément fini mixte bidimensionnel à 7 nœuds avec 5 nœuds déplacement et 2 nœuds contrainte. Cet élément assure la continuité des vecteurs déplacement et contrainte sur la partie cohérente et la discontinuité de celle-ci sur la partie fissurée. Les résultats obtenus, avec l'élément d'interface présenté, montrent une bonne concordance avec les solutions analytiques pour les poutres sandwich.

Abstract:
A mixed interface finite element based on Reissner's mixed variational principle has been presented to analyze sandwich beams. The present element is a 7-node two dimensional mixed finite element with 5 displacement nodes and 2 stress nodes. The mixed interface finite element ensures the continuity of stress and displacement vectors at the interface between two materials. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions for sandwich beams.

Keywords: Mixed finite element, Interface, Reissner's mixed variational principle, sandwich beams.

1 Introduction

Evaluation of the stresses to the perfectly coherent interfaces requires the respect of continuity conditions to the interfaces: continuity of stress and displacement vectors. In the analysis of the structures presenting interfaces such as the laminated composite and the sandwich beams, this evaluation presents at the same time a great interest and a particular difficulty.
In this paper, a mixed interface finite element presented by Bouziane and al. [1] has been used to analyze sandwich beams. The element consists on a 7-node two dimensional mixed finite element developed by using Reissner's mixed variational principle.
The interface element, of 5 displacement nodes with two degrees of freedom (two displacement components u_1, u_2) per node and 2 stress node with two degrees of freedom (two transverse stress components σ_{22} , σ_{12}) per node, ensures the continuity of transverse stress and displacement fields. A brief discussion of the formulation of the mixed interface element has been presented in the next section. Finally, the validity of the present mixed-interface element is demonstrated through some numerical results for a sandwich beam.

2 Mixed finite element

A laminated composite beam consisting of N-layers of lamina has been considered for finite element analysis. The beam has discredized using a 7-node mixed–interface finite element (RMQ-7 element: **R**eissner **M**odified **Q**uadrilateral) as shown in figure 1(a).
The RMQ-7 interface element is a quadrilateral mixed element with 7 nodes and 14 degrees of freedom [1]. This interface element is obtained by successively exploiting the technique of delocalization [2] and the static condensation procedure [3].
Three of its sides are compatible with linear traditional elements and present a displacement node at each corner (figure 1(b)). The fourth side, in addition to its two displacement nodes of corner (node 1

and node 2), offers three additional nodes: a median node (node 5) and two intermediate nodes in the medium on each half-side (nodes 6 and 7), introducing the components of the stress vector along the interface. The Continuity of the displacement and stress vectors can be taken into account on the level on this particular side, which must be placed along the interface.

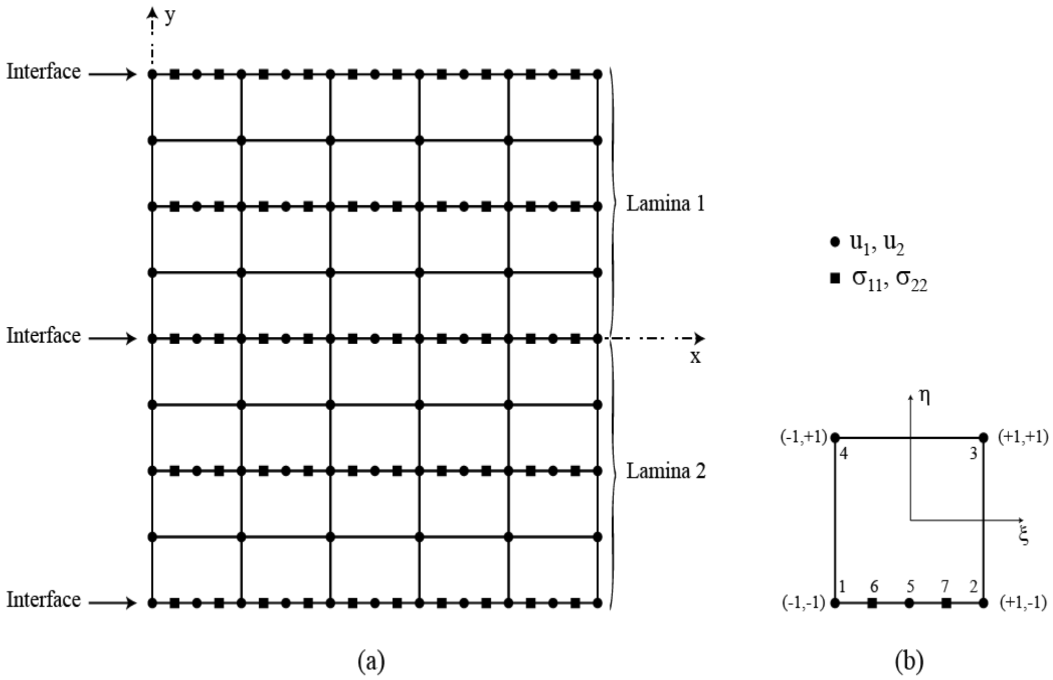

FIG. 1- (a) Discretization of laminated composite beam (b) RMQ-7 element

In the configuration of figure 1(b), the shape functions, used to approximate σ_{22} and σ_{12} are given as follows**:**

$$M_{i2}^{6} = \frac{1}{6}(1-2\xi)(1-2\eta),\ M_{i2}^{7} = \frac{1}{6}(1+2\xi)(1-2\eta)$$
$$M_{i2}^{8} = \frac{1}{6}(1-2\xi)(1-2\eta),\ M_{i2}^{9} = \frac{1}{6}(1+2\xi)(1-2\eta)\ , i = 1, 2 \quad (1)$$

The element matrix $[K_e]$ is given by

$$[K_e]=\begin{bmatrix} [K_{\sigma\sigma}] & [K_{\sigma u}] \\ [K_{\sigma u}] & [0] \end{bmatrix} \quad (2)$$

Here $[K_{\sigma\sigma}] = -e\int_{A_e}[M]^t[S][M]\, dA^e$

and $[K_{\sigma u}] = e\int_{A_e}[M]^t[B]\ \ dA^e$

Where e is the thickness, [S]is the compliance matrix, [M] is the matrix of interpolation functions for stresses, [B]is the strain-displacement transformation matrix and A^e is the element area.

3 Numerical examples

3.1 Example 1

Simply supported sandwich beam has been considered. This beam presents three isotropic layers and presenting coherent interfaces. A sandwich beam, with dimensions and material properties as shown in figure 2 [4], is subjected to uniform load and the interest is primarily centered on the study of transverse shear stresses and the deflection.
To see the convergence rapidity of the transverse shear and deflection several meshes are used. Results obtained through the present mixed element for various numbers of degrees of freedom are tabulated in tables 1 and 2 where they have been compared with the elastic solutions given by Pagano [5]. Table 1 shows the deflection values obtained at x=L/2 according to the number of degrees of freedom. Variation of transverse shear at x = L/4 has been presented in table 2.

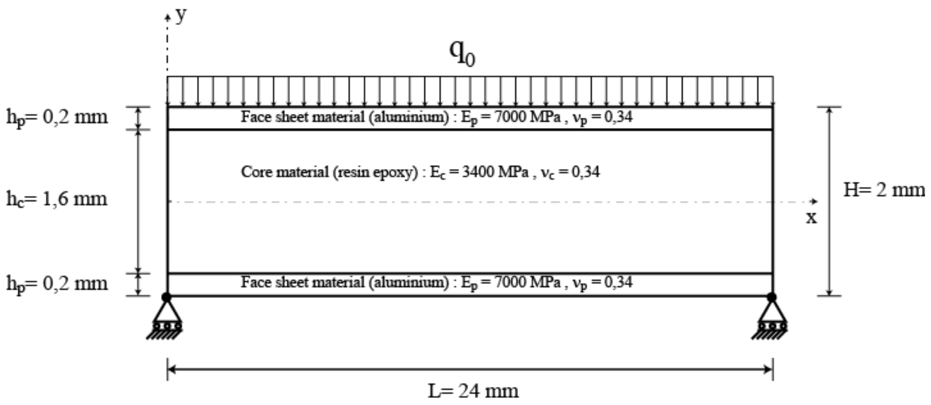

Fig. 2- Sandwich beam analyzed

Element type	Number of degree of freedom	Deflection u_2 (mm)
Present mixed element	32	-0.105
	98	-0.200
	402	-0.209
Pagano [5]	-	-0.208

Table 1. Deflection in a sandwich beam at x=L/2 solved by various meshes

Element type	Number of degree of freedom	σ_{12} Transverse shear (MPa)		
		$y = -h_c/2$	$y = 0$	$y = h_c/2$
Present mixed element	32	-3.026	-	-3.001
	98	-3.135	-	-3.258
	402	-3.197	-3.314	-3.184
Pagano [5]	-	-3.159	-3.431	-3.158

Table 2. Transverse shear in a sandwich beam at x=L/4 solved by various meshes

It can be seen that the results from the present mixed element are in very good agreement with the elasticity solution [5].

3.2 Example 2

Symmetric sandwich beam has been analyzed. This beam is subjected to uniform load and presents three layers including two identical orthotropic face sheet and an isotropic core as shown in figure 3. The material properties of the face sheet and core material are:

- ✓ Face sheet material (orthotropic material): E_x=52800 MPa, E_z=39713 MPa, G_{xz}=13200 MPa, ν_{xz}=0,41
- ✓ Core material (isotropic material): E=50 MPa, G=21 MPa, ν=0,2

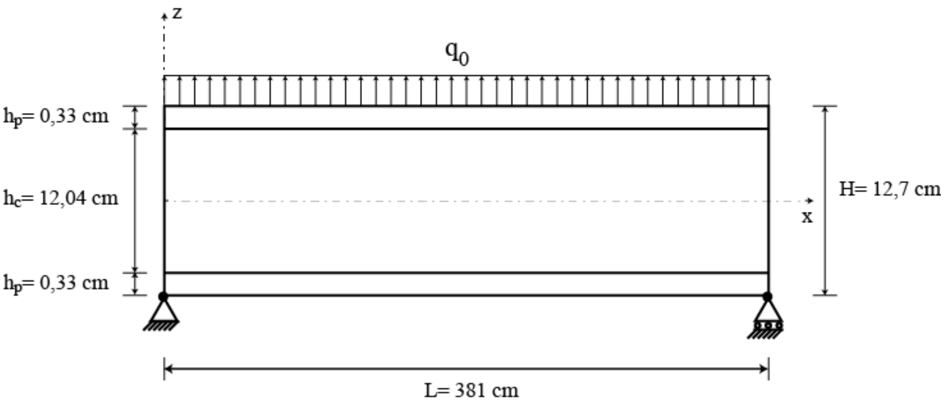

Fig. 3- Sandwich beam analyzed

In order to evaluate the validity and the credibility of the present mixed-interface element, a study of the convergence on a sandwich beam is carried out. In this analysis several meshes are used.
Results obtained through the present mixed element for various numbers of degrees of freedom are tabulated in tables 3, 4 and 5 where they have been compared with the elastic solutions given by Pagano [5]. Table 3 shows the deflection values obtained at x=L/2 according to the number of degrees of freedom. Table 4 gives the horizontal displacement at x=L/4. Variation of transverse shear at x = L/8 has been presented in table 5.

Element type	Number of elements	Number of degree of freedom	Deflection u_2 (cm)
Present mixed element	4	32	1.72
	16	98	2.65
	64	402	2.70
Pagano [5]		-	2.73

Table 3. Deflection in a sandwich beam at x=L/2 solved by various meshes

Element type	Number of degree of freedom	Horizontal displacement u_1 (cm)	
		$z = -h_c/2$	$z = h_c/2$
Present mixed element	32	0.0326	-0.0325
	98	0.0682	-0.0682
	402	0.0690	-0.0690
Pagano [5]	-	0.0705	-0.0705

Table 4. Horizontal displacements in a sandwich beam at x=L/4 solved by various meshes

It appears that the mixed interface element converges very quickly for a number relatively low of degrees of freedom.

Element type	Number of degree of freedom	Transverse shear σ_{12} (MPa)	
		$z = -h_c/2$	$z = h_c/2$
Present mixed element	32	0.1123	0.1123
	98	0.1137	0.1146
	402	0.1131	0.1159
Pagano [5]	-	0.115	0.115

Table 5. Transverse shear in a sandwich beam at x=L/8 solved by various meshes

4 Conclusion

A novel mixed-interface finite element has been used to analyze a sandwich beam. In the formulation of this element, we used Reissner's mixed variational principle to build the parent element. The mixed interface finite element is obtained by successively exploiting the technique of delocalization and the static condensation procedure. This element ensures the continuity of transverse stress and displacement fields at the interface. The accuracy of the element has been evaluated by comparing the numerical solution with an available analytical solution. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions. Comparison of the results with the exact elastic solutions validates the present interface element and indicates its application to sandwich beams problems.

References

[1] Bouziane S., Bouzerd H and Guenfoud M., Mixed finite element for modelling interfaces, European Journal of Computational Mechanics, 18(2), 155-175, 2009.

[2] Verchery G., Méthodes numériques de calcul des champs de contraintes dans les matériaux hétérogènes, Calcul des Structures et Intelligence Artificielle, Fouet J. M., Ladeveze P., Ohayon R., vol. 1, Paris, Pluralis, 7-21, 1987.

[3] Gallagher R. H., Introduction aux éléments finis, Paris, Pluralis (traduction française), 1976.

[4] Aivazzadeh S., Eléments finis d'interface. Applications aux assemblages collés et structures statifiées, thèse de docteur ingénieur de l'université de technologie de Compiègne, 1984.

[5] Pagano N. J., Exact solutions for composite laminates in cylindrical bending, Journal of Composite Materials, 3, 398-411, 1969.